

A HYBRID GHOST FLUID – EXTENDED FINITE ELEMENT METHOD

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Recent publications describe methods for embedding interfacial jumps within finite difference[1] and finite element methods[2]. These methods seek to decouple the interfacial motion from the mesh. Using these methods, the moving interfaces found in multiphase problems have been simulated using a fixed mesh. This requires methods for embedding interfacial discontinuities within a finite difference stencil or a finite element.

At first glance it appears that the methods used by finite difference and finite element practitioners are quite different. A class of finite difference methods has been termed “Ghost Fluid” methods[1]. Normally, a finite difference stencil for a node close to the interface would incorporate nodal values from both sides of the interface. This causes unphysical solutions, however, when a discontinuity cuts through this stencil. The discontinuity violates the finite difference assumption that the derivatives are continuous within the stencil. Ghost Fluid methods resolve this by extrapolating nodal values that are consistent with each side of the interface. Instead of sampling values from both sides of the interface, values that would normally come from the other side of the interface are replaced with these consistent extrapolated values. The discontinuity is effectively removed from the finite difference operators, and the discontinuity is captured.

Concurrent with these developments, new algorithms have been proposed for embedding interfacial discontinuities in finite element methods. A class of these methods has been termed extended finite element methods (XFEM)[2]. In this approach the elements near the interface are augmented with additional degrees of freedom that can accommodate the interfacial jumps. Depending on the application, the constraint equations for these additional unknowns can be derived from the normal Galerkin method, may involve additional penalized conditions, or may incorporate additional Lagrange multipliers.

In this talk, the interrelation between ghost fluid and extended finite element methods is described. One advantage of typical ghost fluid methods is that the nodal values are modified rather than the structure of the finite difference stencil. A similar method is described for finite element methods where the element assembly is changed very little but the nodal values are manipulated instead. These methods are explored for the case of a quadratic, 1-dimensional finite element applied to energy equation with an embedded jump in the temperature gradient.

References

- [1] R. Fedkiw, T. Aslam, B. Merriman, and S. Osher, “A Non-Oscillatory Eulerian Approach to Interfaces in Multimaterial Flows (The Ghost Fluid Method),” *Journal of Computational Physics*, v. 152, p. 457-492, 1999.
- [2] J. Chessa, P. Smolinski, and T. Belytschko, “The Extended Finite Element Method (XFEM) for Solidification Problems,” *International Journal for Numerical Methods in Engineering*, v. 53, p. 1959-1977, 2002.